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Principles of Economics and Management I

Session 5 · Market Equilibrium and Taxes

Introduction



Where We Came From

Session 2 and 3 gave us **demand** and its elasticity; session 4 gave us **supply** as marginal cost.

Today the two curves finally meet, and a single number comes out of it: the market **price**. 

Then we look at what a **tax** does, and who really pays it. 



Today's Map

1.  Equilibrium, and why nobody has to know the price in advance
2.  Total surplus, and why the equilibrium maximizes it
3.  What happens when the price is wrong
4.  Shifts, the four step process and the four cases
5.  Application: the credit market and the interest rate
6.  Taxes: the wedge, incidence and deadweight loss

Two interactive widgets today. Bring a mouse. 

Part 1 · Equilibrium and Total Surplus



Bringing the Two Sides Together

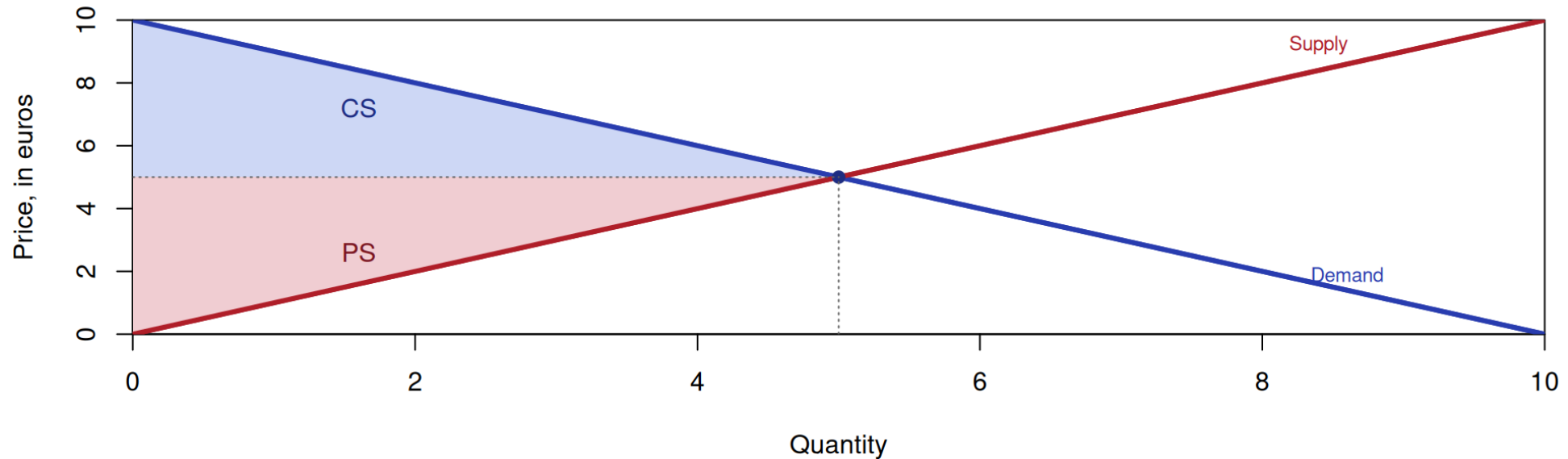
Demand is the sorted list of **values**. Supply is the sorted list of **marginal costs**.

Both are functions of the same price. So ask the only question left: at which price do the two quantities **agree**?

Equilibrium: the price p^* at which $Q_d(p^*) = Q_s(p^*)$.



Equilibrium



Consumer surplus above the price, producer surplus below it.
Both already defined, now in one picture.

✨ Total Surplus

Total surplus (TS): consumer surplus plus producer surplus.
What the market as a whole gains from trading.

$$TS = CS + PS = \sum_i (v_i - p) + \sum_j (p - c_j) = \sum (v_i - c_j)$$

The price **cancels**. It decides how the gains are split. It does not change how big the total is.

 Source: OpenStax *Micro* 3e, §3.5 “Demand, Supply, and Efficiency”, where $CS + PS$ is called **social surplus**.

Why the Equilibrium Maximizes It

Trade unit by unit, highest value buyer against lowest cost seller. Each trade adds $v - c$ to the total.

Below Q^* : some buyer still has $v > c$ of some idle seller. A gain is being left on the table.

Above Q^* : the next trade would need $v < c$. It **destroys** value.

So the total stops rising exactly where $v = c$, which is Q^* . A marginal condition again, just like session 1.

Say it carefully: the equilibrium maximizes the **size** of the pie. It says nothing about whether the split is fair. ⚠️

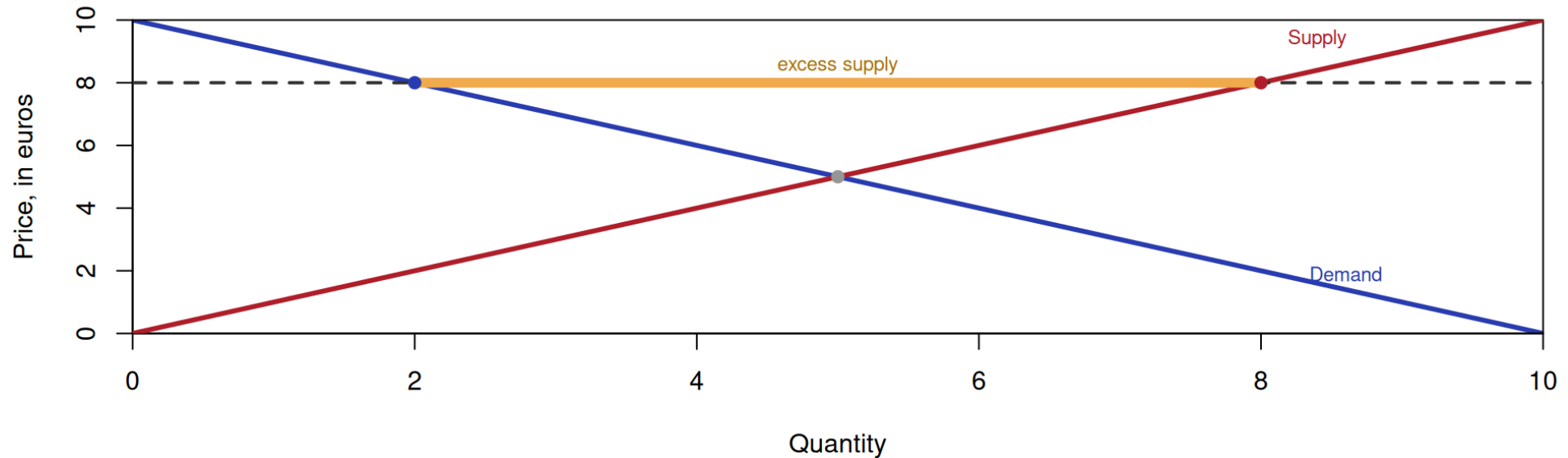
What if the Price Is Wrong?

Nobody in the market is told what p^* is. So suppose the price is simply **wrong**, and watch what happens.

There are only two ways to be wrong: too high, or too low. 



Too High: Excess Supply



At 8 euros sellers offer 8 units and buyers want 2. **Six units go unsold.**

▼ The Pressure That Follows

Unsold stock is expensive: storage, spoilage, capital tied up in a warehouse.

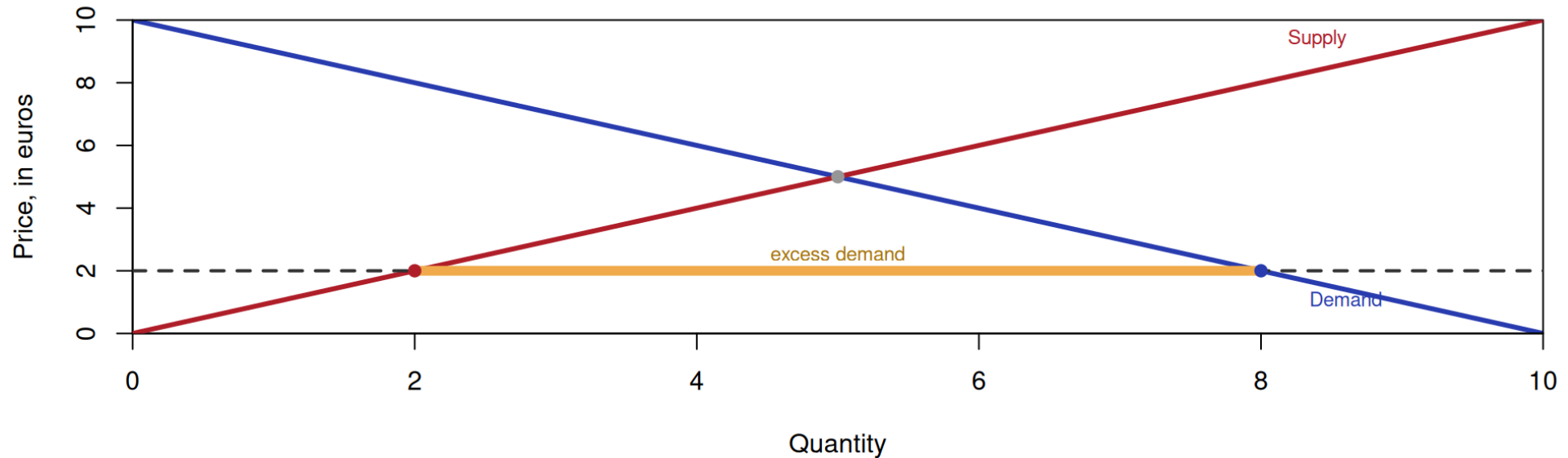
Any single seller who shaves the price sells **first**. So somebody always shaves it. 

The price falls, the orange gap narrows, and it keeps falling while any gap remains.

 Source: OpenStax *Micro* 3e, §3.1, “Equilibrium: Where Demand and Supply Intersect”, which calls this excess supply, or a **surplus**.



Too Low: Excess Demand



At 2 euros buyers want 8 units and sellers offer 2. **Six buyers go home empty-handed.**

▲ The Pressure That Follows

Some of those buyers value the good far above 2 euros. Offering a bit more beats going without.

And sellers notice they can raise the price and **still** sell everything they have. 

The price rises until the queue disappears.

 Source: OpenStax *Micro* 3e, §3.1, “Equilibrium: Where Demand and Supply Intersect”, which calls this excess demand, or a **shortage**.

The Equilibrium Is a Magnet

Write the adjustment as a rule: the price moves in the direction of the gap,

$$\dot{p} = \lambda [Q_d(p) - Q_s(p)], \quad \lambda > 0$$

Q_d is decreasing in p and Q_s is increasing, so the bracket is **strictly decreasing**: positive below p^* , negative above it.

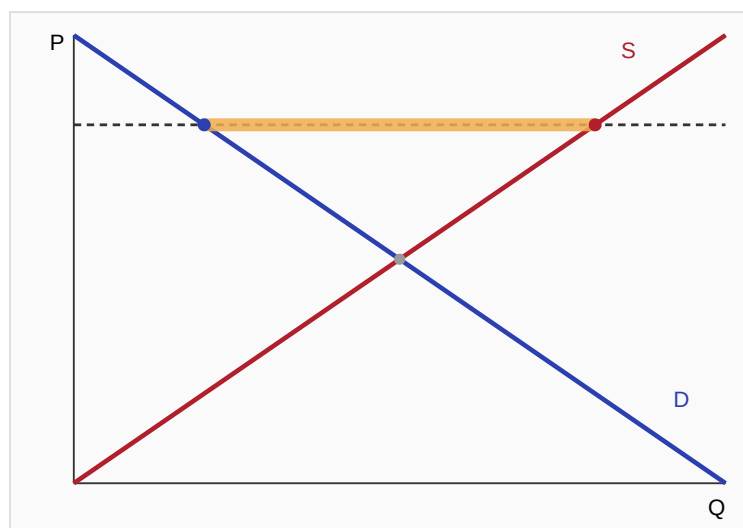
That is the condition for p^* to be a **stable** rest point. Note that nobody in the market needs to know p^* for this to work. 🧭

This uses that demand slopes down and supply slopes up. Without it the bracket does not need to be decreasing, and p^* can be unstable.



Try It: Set the Price Yourself

Drag the price. Watch the two quantities separate, and read off which way the pressure pushes. Only one price leaves no gap.



Price: **8.00** euros



Quantity demanded = **2.00**

Quantity supplied = **8.00**

Excess supply of 6.00. The price is pushed DOWN.

? Multiple Choice 1

At a price **above** the equilibrium:

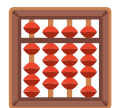
- A. There is excess demand and the price is pushed up.
 - B. The market is at rest, since sellers are happy.
 - C. The demand curve shifts to the left.
 - D. There is excess supply and sellers face pressure to cut the price.
-  **D.** Above p^* the quantity supplied exceeds the quantity demanded.

? Multiple Choice 2

Total surplus at the equilibrium does **not** depend on:

- A. The values buyers place on the good.
- B. The price at which the market clears.
- C. The marginal costs of the sellers.
- D. The quantity traded.

✓ **B.** In $TS = \sum (v_i - c_j)$ the price cancels: it moves surplus between buyers and sellers without changing the total. The other three all enter the sum directly.



Numerical Question

Demand $P = 10 - Q$ and supply $P = Q$, with P in euros. The industry also pays a fixed cost of 8 euros in total.

- a) Find the equilibrium price and quantity.
- b) Compute consumer surplus.
- c) Compute producer surplus.
- d) Compute total surplus.
- e) Compute industry profit, and explain the difference from (c).



Solution

- a)** $10 - Q = Q \Rightarrow Q^* = 5$ and $P^* = 5$ euros.
- b)** CS is the triangle between demand and the price: $\frac{1}{2} \times 5 \times (10 - 5) = 12.5$ euros.
- c)** PS is the triangle between the price and supply: $\frac{1}{2} \times 5 \times (5 - 0) = 12.5$ euros.
- d)** $TS = 12.5 + 12.5 = 25$ euros. Check: $\int_0^5 [(10 - q) - q] dq = 25$.
- e)** Profit = PS - FC = 12.5 - 8 = 4.5 euros. PS ignores the fixed cost; profit does not, exactly as session 4 said.

Part 2 · Comparative Statics

↔ Moving Along vs Shifting

Now the curves themselves move. Watch out for the classic confusion:

A change **in the price of the good** moves us **along** the same curve. The price is an axis, so it cannot move the curve.

A change in **anything else** (income, tastes, costs, technology) **shifts** the whole curve.

Curves **expand** and **contract**. Avoid saying “demand rises”: it could mean either of the two.



What Shifts Demand



Income: more income expands demand, for a normal good. For an inferior good it contracts. That is income elasticity from session 3, with a sign.



Tastes and fashion: the product becomes popular and demand expands.



The price of other goods: a dearer substitute expands demand, a dearer complement contracts it. Cross-price elasticity, again with a sign.



Number of buyers and  **expected future prices.**

 Source: OpenStax *Micro* 3e, §3.2, "What Factors Affect Demand?", which runs through the same list one shifter at a time.



What Shifts Supply



Input costs: dearer raw materials, wages or energy contract supply.



Technology: better technology expands supply, because marginal cost falls.



External conditions: weather, regulation, the number of firms in the market.

Anything that changes **marginal cost** shifts supply. Session 4 told you why: supply **is** marginal cost.

 Source: OpenStax *Micro* 3e, [§3.2](#), “How Production Costs Affect Supply” and “Other Factors That Affect Supply”.



The Four Step Process

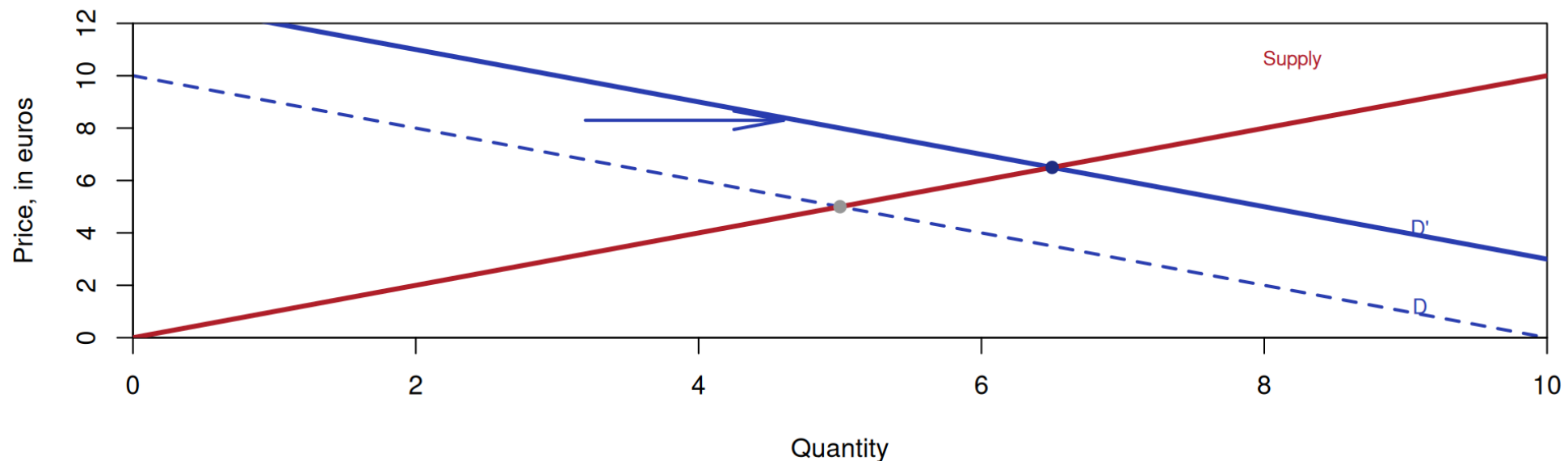
A recipe for predicting any market change:

1. Draw supply and demand **before** the change, and mark the initial equilibrium.
2. Decide: does the change hit **demand** or **supply**?
3. Decide the **direction**: does that curve expand or contract?
4. Compare the **new** equilibrium with the old one.

 Source: OpenStax *Micro* 3e, §3.3 “Changes in Equilibrium Price and Quantity: The Four-Step Process”, where the recipe is applied to a summer of unusually good weather for salmon fishing.



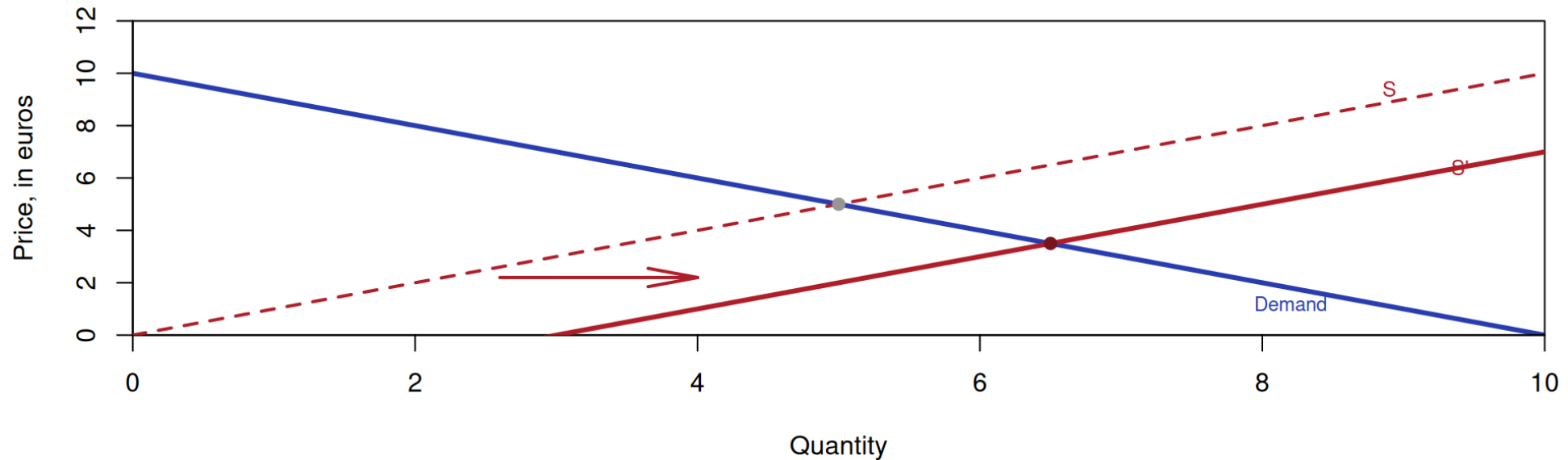
Demand Expands



Supply is unchanged, so we slide **up along** it: the price **rises** and the quantity **rises**.



Supply Expands



Demand is unchanged, so we slide **down along** it: the price **falls** and the quantity **rises**.



The Four Cases

Shock	P^*	Q^*
Demand expands	rises	rises
Demand contracts	falls	falls
Supply expands	falls	rises
Supply contracts	rises	falls

Rule of thumb: with a **demand** shock, price and quantity move **together**. With a **supply** shock, they move **opposite**.

The reverse reading works too: price and quantity moving together in the data suggests **demand** moved. 🔍

And if Both Shift at Once?

If demand and supply move at the same time, one variable is **determined** and the other is **ambiguous**.

Both expand: the quantity rises for certain, the price can go either way. It depends on **which shift is larger**.

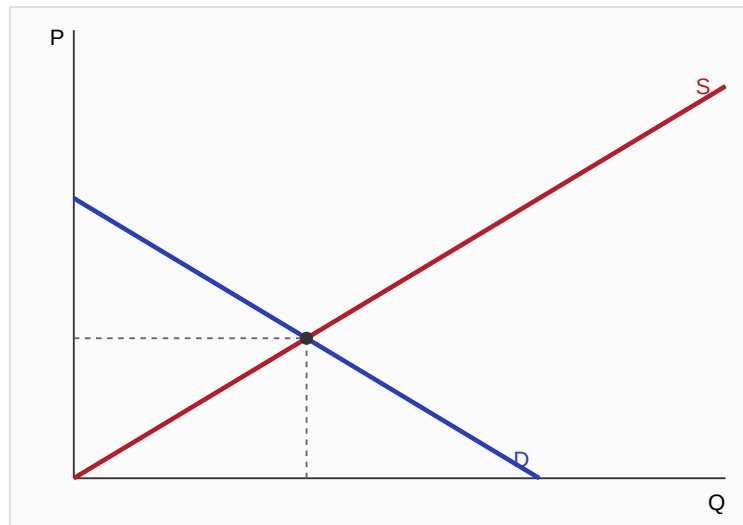
Demand expands while supply contracts: the price rises for certain, the quantity is ambiguous.

To settle the ambiguous one you need the numbers: how large each shift is. 



Try It: Move the Curves

Shift demand and supply, one at a time, and check the four cases. Then move both together and watch one of the two answers become ambiguous. 



Demand shift: 0



Supply shift: 0



$P^* = 5.00$ $Q^* = 5.00$

P^* unchanged, Q^* unchanged



Example: A Good Coffee Harvest

An exceptional harvest at origin makes coffee cheaper to produce.

Step 2: it hits **supply**, because it changes the cost of producing.

Step 3: supply **expands**, the curve shifts right.

Step 4: the price **falls** and the quantity **rises**. 

 The mirror image is in OpenStax *Micro* 3e, [§3.5](#): a terrible frost hits the Brazilian coffee crop, supply contracts and the price rises. Same market, opposite shock.



Example: A Change in Tastes

Consumers start to prefer tea over coffee. What happens in the coffee market?

Step 2: it hits **demand**, since it is a change in tastes.

Step 3: demand for coffee **contracts**, the curve shifts left.

Step 4: the price **falls** and the quantity **falls**. Both together, as the four cases predict. 

 Source: OpenStax *Micro* 3e, [§3.2](#), "Changing Tastes or Preferences".

Example: An Energy Price Shock

Energy prices in Europe jump. What happens in a market where energy is a large input?

Step 2: supply, through marginal cost. **Step 3: supply contracts.**

Step 4: the price rises and the quantity **falls**.

That combination has a name you will meet in Block 2: a **supply shock**, and it is why inflation and weak output can arrive together.





Application: The Interest Rate Is a Price

Here is the central application for anyone heading into finance:

The **interest rate** is the price of money over time: the price of using someone else's funds today.

Like any price, it comes out of the meeting of a **supply** and a **demand**. Everything from Part 1 and Part 2 applies unchanged.





Who Supplies and Who Demands Funds

■ **Supply of funds (saving):** those with savings, lending them out. A higher rate makes saving more attractive, so the curve slopes **up**.

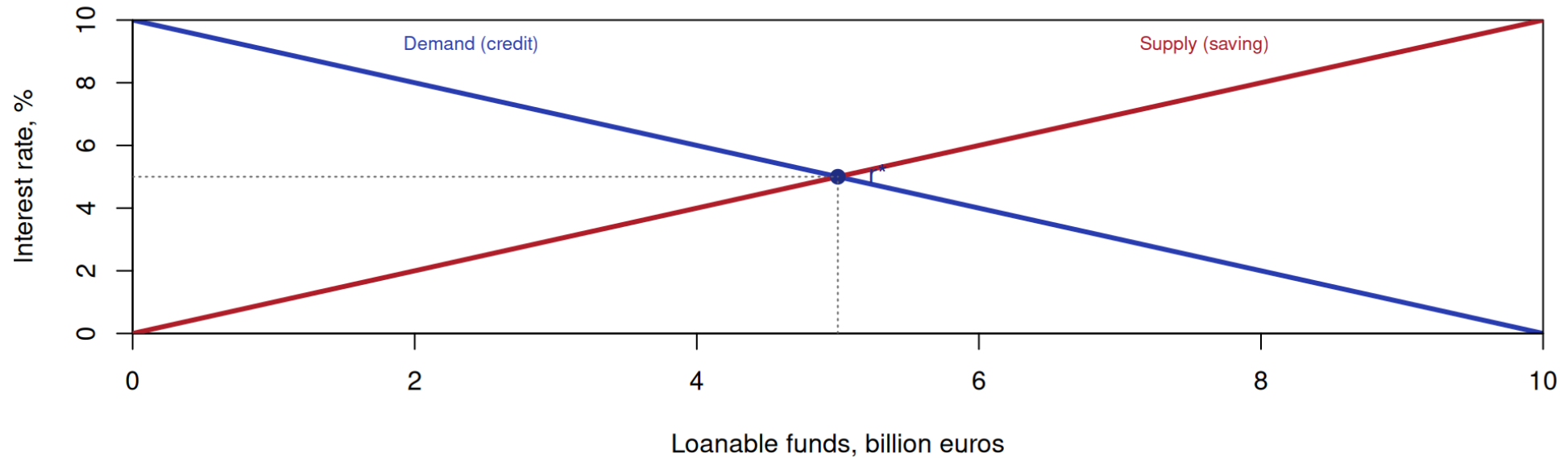
■ **Demand for funds (credit):** firms and households seeking financing. A higher rate kills the marginal project, so the curve slopes **down**.

Same reservation-price logic: a firm borrows while the return on the project **exceeds** the rate. The demand for funds is the sorted list of project returns.

 Source: OpenStax *Micro* 3e, §4.2 “Demand and Supply in Financial Markets”, which runs the argument through the market for credit card borrowing.



Equilibrium in the Credit Market



The equilibrium rate r^* is what balances saving against borrowing.

↔ Comparative Statics in Credit

Firms turn optimistic and want to invest more. Run the four steps.

Demand for credit expands, shifting right. Saving is unchanged.

So the **rate rises** and the quantity of funds traded **rises**. Price and quantity together: a demand shock, exactly as the table said.



And a savings glut?

Supply expands, so the **rate falls** and the quantity still rises.

Opposite directions: a supply shock.

 Source: OpenStax *Micro* 3e, §4.2, “Shifts in Demand and Supply in Financial Markets”.



The Mirror Image: Bond Prices

A bond paying coupons C_t and maturing at T is worth, today,

$$P = \sum_{t=1}^T \frac{C_t}{(1+r)^t}$$

Every single term falls when r rises, so $\frac{dP}{dr} < 0$, always. The bond price and the interest rate move in **opposite** directions.

So when the credit market pushes r^* up, the price of every bond already issued falls. 

The ECB moves these rates directly, which is why a room full of people watches a press conference. We return to it in **monetary policy**, session 11. 

? Multiple Choice 3

Demand and supply **both expand** at the same time. Then:

A. The quantity rises for certain, and the price can go either way.

B. The price rises for certain, and the quantity can go either way.

C. Both the price and the quantity rise for certain.

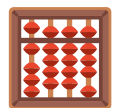
D. Nothing at all can be said.

 **A.** Both shifts push the quantity up, so that one is safe. On the price they pull in **opposite** directions, so the answer depends on which shift is larger. Note that D is too pessimistic: one of the two is always pinned down.

? Multiple Choice 4

Interest rates rise. Other things equal, the price of a bond already issued, with a fixed coupon:

- A. Rises, because the bond now pays more.
 - B. Does not change, because the coupon is fixed.
 - C. Falls, because its fixed flows are discounted more heavily.
 - D. Rises or falls, depending on the issuer.
- ☒ **C.** The coupon being fixed is exactly the point: the flows do not move, but the discount factor does. Every term of the sum shrinks.



Numerical Question

Credit market: demand $r = 12 - Q$ and supply $r = 2 + Q$, with r in percent and Q in billion euros.

- a) Find the equilibrium rate and quantity.
- b) Firms turn optimistic and demand becomes $r = 15 - Q$. Find the new equilibrium.
- c) Which of the four cases is this, and does the answer match?
- d) What happens to the price of bonds already issued?



Solution

- a) $12 - Q = 2 + Q \Rightarrow 10 = 2Q$, so $Q^* = 5$ billion euros and $r^* = 7$ percent.
- b) $15 - Q = 2 + Q \Rightarrow 13 = 2Q$, so $Q^* = 6.5$ billion euros and $r^* = 8.5$ percent.
- c) Demand expanded, so price and quantity should move **together**. Both rose. 
- d) The rate rose, so existing bond prices **fall**: the same fixed flows, discounted at a higher rate. 

Part 3 · Taxes



The Incidence of a Tax

The state levies a per unit tax. Who actually pays it, the buyer or the seller?

Incidence is decided by the **elasticities**. Who hands the tax to the state does not matter.

The **more inelastic** side bears most of the tax.

 Source: OpenStax *Micro* 3e, §5.3, "Elasticity and Tax Incidence", worked through with a tax on cigarettes.

The Tax Wedge

A per unit tax t drives a **wedge** between the two prices:

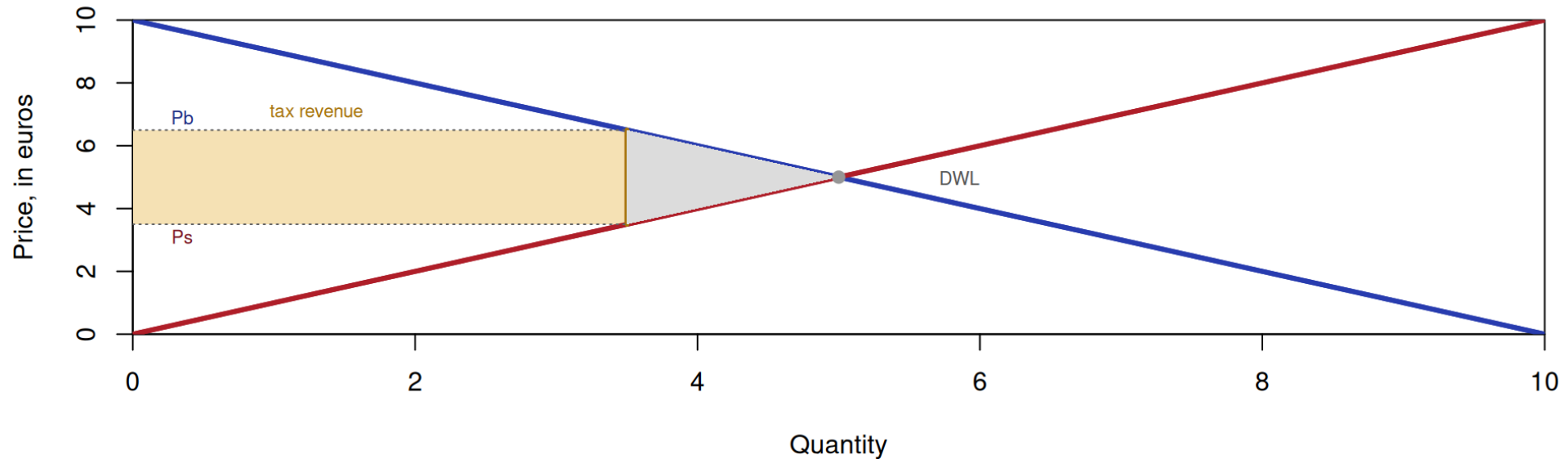
$$P_b = P_s + t$$

The buyer pays P_b , the seller keeps P_s , and the state takes the difference. The market clears at $Q_d(P_b) = Q_s(P_s)$.

Nothing in these two equations mentions **who hands the money over**. That is why the legal side of the tax is irrelevant to who really pays.



The Tax Wedge



The tax shrinks the quantity traded. The shaded rectangle goes to the state; the grey triangle goes **nowhere**.



The Incidence Rule

The share of the tax borne by consumers is

$$\frac{\varepsilon_s}{\varepsilon_s + |\varepsilon_d|},$$

and the share borne by producers is the rest, $\frac{|\varepsilon_d|}{\varepsilon_s + |\varepsilon_d|}$.

The larger the elasticity of one side, the smaller its share: that side can walk away from the trade more easily.

Two limits to sanity check it. Perfectly inelastic demand ($\varepsilon_d = 0$): the share is 1, consumers pay all of it. Perfectly elastic demand: the share is 0, producers pay all of it.

Very inelastic demand (tobacco, fuel): almost the whole tax falls on the **consumer**. This is one reason governments like to tax them. 🧐



The Incidence Rule: Proof

Start at the equilibrium (P^*, Q^*) with no tax, and add a small tax t . The market clears when

$$Q_d(P_s + t) = Q_s(P_s).$$

This defines P_s as a function of t . Differentiate both sides with respect to t :

$$Q'_d \left(\frac{dP_s}{dt} + 1 \right) = Q'_s \frac{dP_s}{dt} \Rightarrow \frac{dP_s}{dt} = \frac{Q'_d}{Q'_s - Q'_d}.$$

And since $P_b = P_s + t$,

$$\frac{dP_b}{dt} = 1 + \frac{dP_s}{dt} = \frac{Q'_s}{Q'_s - Q'_d}.$$

With $Q'_d < 0 < Q'_s$, both are between 0 and 1 in absolute value, and $\frac{dP_b}{dt} - \frac{dP_s}{dt} = 1$: each euro of tax is split between the two sides.



The Incidence Rule: Proof

Now multiply numerator and denominator by P^*/Q^* :

$$\frac{dP_b}{dt} = \frac{Q'_s \frac{P^*}{Q^*}}{Q'_s \frac{P^*}{Q^*} - Q'_d \frac{P^*}{Q^*}} = \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_d} = \frac{\varepsilon_s}{\varepsilon_s + |\varepsilon_d|}.$$

The elasticities from session 3, evaluated at the equilibrium. ■

For a small tax this is exact. With **linear** curves the slopes are constant, so it holds for any t . Otherwise it is an approximation.

Check it on the numerical question below: at $P^* = 5$, $Q^* = 10$ you get $\varepsilon_d = -2 \cdot \frac{5}{10} = -1$ and $\varepsilon_s = 1$, so consumers pay half. 🎲



Deadweight Loss

The rectangle is a **transfer**: buyers and sellers lose it, the state gains it. The triangle is not.

Deadweight loss (DWL): the value of the beneficial trades that no longer happen because a wedge stopped the price from clearing.

For a wedge t that cuts the quantity by ΔQ , the lost triangle is

$$\text{DWL} \approx \frac{1}{2} t \Delta Q$$

Those are trades where $v > c$ and which nobody now makes. Nobody collects that value. 

 Source: OpenStax *Micro* 3e, §3.5, “Demand, Supply, and Efficiency”, which shows the same lost triangle without the algebra.



Why the Loss Grows with the Square

ΔQ is itself roughly proportional to t , so $DWL \approx \frac{1}{2}t \Delta Q$ grows with t^2 .

A tax twice as large costs about **four** times as much in lost surplus. That is the standard argument for many small taxes rather than one large one. 

And the loss grows with the **elasticities**, because ΔQ does. Tax the side that cannot walk away and you raise revenue with less damage.

The problem is that the side that cannot walk away (fuel, food, rent) is often not the side we would like to tax. Efficiency and fairness can point to different taxes. 

So Is Taxing Wrong?

No. Efficiency is one criterion, not the only one. Distribution, insurance and public goods are real objectives, and the market does not deliver any of them by itself.

What the analysis buys you is the **price tag**: this is what the objective costs in forgone surplus, and here is who pays it.

? Multiple Choice 5

A per unit tax is levied on sellers rather than buyers. Compared with levying it on buyers, the share actually borne by consumers:

- A. Is smaller, since sellers hand over the money.
 - B. Is exactly the same.
 - C. Is larger, since sellers pass everything on.
 - D. Cannot be determined without knowing the tax rate.
- ✓ **B.** The wedge $P_b = P_s + t$ and the clearing condition never mention who pays the state. Incidence is settled by the two elasticities alone.

? Multiple Choice 6

A government doubles a per unit tax. The deadweight loss, roughly:

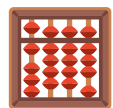
A. Stays the same, since the market is the same.

B. Doubles.

C. Quadruples.

D. Halves, since the quantity traded falls.

✓ C. With $DWL \approx \frac{1}{2}t \Delta Q$ and ΔQ itself proportional to t , the loss grows with t^2 .



Numerical Question

Demand $Q_d = 20 - 2P$ and supply $Q_s = 2P$, with P in euros. A per unit tax of $t = 2$ euros is imposed.

- a) Find the equilibrium before the tax.
- b) Find P_b , P_s and Q after the tax.
- c) How much revenue does the state collect?
- d) Compute the deadweight loss.



Solution

- a)** $20 - 2P = 2P \Rightarrow P^* = 5$ euros and $Q^* = 10$.
- b)** Clearing needs $20 - 2P_b = 2P_s$ with $P_b = P_s + 2$. Substituting, $20 - 2P_s - 4 = 2P_s$, so $P_s = 4$, $P_b = 6$ and $Q = 8$.
- c)** $t \times Q = 2 \times 8 = 16$ euros. Note the tax split evenly, one euro each way: here the two elasticities are equal in size.
- d)** $\frac{1}{2} \times t \times \Delta Q = \frac{1}{2} \times 2 \times (10 - 8) = 2$ euros of surplus destroyed.

Wrap-Up

What to Take From This Session

 **Equilibrium** is where quantity demanded equals quantity supplied. Away from equilibrium, the price moves toward the side with the gap.

✨ **Total surplus** is $\sum (v_i - c_j)$, and the price does not appear in it. The equilibrium maximizes total surplus. It does not determine how that surplus is divided between buyers and sellers.

↔ Demand shocks move price and quantity **together**. Supply shocks move them in **opposite** directions. The four step process handles any shock; when both curves shift at once, one of the two outcomes stays ambiguous.

 The **interest rate** is the price of funds. Bond prices move in the opposite direction from the interest rate.

 A tax drives a **wedge** between the price buyers pay and the price sellers receive. The more **inelastic** side bears most of the tax, regardless of which side pays the state directly. Deadweight loss grows with t^2 .

Next Session

Uncertainty, Risk and Information.

Every price so far was known with certainty. Next we drop that: expected utility, risk aversion, and what happens when one side of a trade knows more than the other. 

See you next week. 